Universal witness of classical correlations for two-photon states

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The threshold between classical and nonclassical two-qubit states is drawn at the place when those states can no longer be described by classical correlations, i.e., quantum discord or entanglement appear. However, to check if the correlations are classical (in terms of quantum discord and entanglement) it is sufficient to witness the lack of quantum discord because its zero value implies the lack of entanglement. In this Rapid Communication we introduce a quantum discord witness which is able to detect quantum discord for a wide variety of states including symmetric states and is a universal classicality witness up to local unitary transformations. We also discuss how this witness can be practically measured in linear-optical systems using standard beam splitters and photon detectors. We study the efficiency of the setup assuming both ideal and real components and show that the efficiency of the proposed implementation is better than the full two-photon quantum tomography. We expect that a similar approach to the presented one can be used to directly measure quantum discord. The variance-like mathematical form of the discovered witness can be a manifestation of a nontrivial physical phenomenon possibly related to quantum phase transitions.

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Introduction. The threshold between the classical and quantum world fascinates physicist since the discovery of quantum phenomena and realizing how different they are form our everyday experience. One of the prominent examples of quantum behavior is the non-locality leading to violation of Bell's inequalities [1, 2]. For two-level systems there is no non-locality without quantum entanglement, but the opposite can be true [3]. Quantum entanglement plays a prominent role in quantum information processing [4]. However, the entanglement is not the only type of non-classical correlations. As described by Ollivier and Zurek [5] the non-classical correlations can be associated with *quantum discord*. Quantum discord (QD) is useful in many ways including quantum information processing or detection of quantum phase transitions, especially in the cases when the entanglement fails to grasp this phenomenon [6]. Moreover, it was demonstrated that only classical correlations can be broadcast locally [7]. All this features of quantum discord motivate the quest for developing tools for detecting and quantifying it. Nevertheless, there were only three experimental implementations of witnesses of classical correlations, or *classicality witnesses* (CWs), in discrete variable systems. Two of them were implemented in nuclear magnetic resonance systems [8, 9] and one using linear optics [10], however these witnesses were not universal. At this point, we should stress that detecting purely classical correlations is a difficult problem since it involves solving optimization problem over not convex set of classical states. Thus, the problem of detecting classical correlations is harder that detection of entanglement. Moreover, any CW should be nonlinear [11]. For those reasons the CWs [10–12] are usually non-universal. However, there Zhang *et al.*[13] demonstrated that finding an universal witness for classical correlations is possible, but the established witness is not suitable for optical implementation. In this Rapid Communication we introduce CW which overcomes the limitations of all the previously developed witnesses.

Let us start with introducing some basic definitions used throughout our Rapid Communication. A general two-qubit density matrix ρ can be expressed in the Bloch representation as

$$\rho = \frac{1}{4}(I\otimes I + \vec{x}\cdot\vec{\sigma}\otimes I + I\otimes\vec{y}\cdot\vec{\sigma} + \sum_{n,m=1}^{3} T_{nm}\,\sigma_n\otimes\sigma_m), \quad (1)$$

where $\vec{\sigma} = [\sigma_1, \sigma_2, \sigma_3]$ and matrix $T_{ij} = \mathrm{Tr}[\rho(\sigma_i\otimes\sigma_j)]$ are given in terms of the Pauli matrices, and $x_i = \mathrm{Tr}[\rho(\sigma_i\otimes I)]$ ($y_i = \mathrm{Tr}[\rho(I\otimes\sigma_i)]$) describe Bloch vector $\vec{x}$ ($\vec{y}$) of the first (second) subsystem, later referred to as A and B. Moreover, it is always possible to transform ρ with local unitary operations [14] so that T becomes a diagonal matrix.

The state ρ is not entangled (is separable) when it has a positive partial transpose, i.e., is a PPT state (see Peres-Horodecki criterion [15, 16]). The lack of entanglement for a two-qubit system implies, e.g., locality, in terms of violation of the Bell-CHSH inequality [2] (for quantitative study see [17]), and thus it corresponds to classical situation where the measurement outcomes can be explained by a hidden-variable model. However, quantum entanglement is not responsible for all the nonclassical effects. One of the recently celebrated manifestation of quantumness is *quantum discord* [5]. The QD is responsible for difference in conditional quantum information calculated by in two ways, where one of them uses the Bayesian rule for calculating probabilities. Therefore, QD quantifies how much conditional quantum probabilities differ

from those calculated within classical theory. When the QD vanishes if the state fulfills the strong PPT condition [18], i.e., ρ has to be PPT and its PPT must admit Cholesky decomposition (there are also other so-called nullity conditions – for review see [6]). Thus, if there is no discord, there is no entanglement. However, the reverse does not have to be true.

There are several ways of quantifying QD. The one for which an analytic formula is known [19] is the so-called *geometric quantum discord* (GQD) quantifying Hilbert-Schmidt distance to the closest non-discordant state. The expression for the GQD reads as

$$D_i(\rho) = -\frac{1}{4}\left(\lambda_{\max,i} - \sum_{n=0}^{2}\lambda_{n,i}\right), \qquad (2)$$

where $\lambda_{n,i}$ (for $i = A, B$) stand for eigenvalues of matrix $K_A = \vec{x}\vec{x}^\dagger + TT^\dagger$ or $K_B = \vec{y}\vec{y}^\dagger + TT^\dagger$, where $\dagger$ denotes transposition. The largest $\lambda_{n,i}$ is denoted as $\lambda_{\max,i}$. Note, that D_i is asymmetric. Thus, if $D_A = 0$ the state is called classical-quantum or if $D_B = 0$ the state is quantum-classical. Naturally, there have been attempts of finding analytic formula for the symmetric GQD, which answers the question about the closest classical-classical state, however this is still an open problem [6, 20].

If $D_A = D_B = 0$ the state is classical-classical since it does not exhibit quantum correlations responsible for discord between conditional quantum information calculated in the two above-mentioned ways. In the following sections we find an experimentally accessible criterion allowing to identify states of zero D_i and describe how to implement it within the framework of linear-optics.

Universal classicality witness. Here we present one of the most important results, i.e., the witness – W_i. It is constructed to fulfill $W_i > 0$, if the investigated states have nonzero GQD, and $W_i = 0$, if the subsystems i is correlated with the other one in classical way. The witness reads as

$$W_i = M_{1,i}^2 - M_{2,i} \geq 0, \qquad (3)$$

where $M_{n,i} = \sum_{m=0}^{2}\lambda_{m,i}^n$ for $n = 1, 2$ are moments of the matrix K_i ($i = A, B$) from Eq. (2), where λ_m denotes m'th eigenvalue of K_i. Note that W_i and K_i are asymmetric, thus W_i cannot exclusively detect classical-classical states. One of possible symmetric CWs is $W_s = W_A + W_B$. For the witness W_i to work K_i matrix has to be positive semidefinite.

To show that our CW fulfills the necessary condition for witnessing one-sided classical correlations let us start with expressing the GQD D_i in terms of moments $M_{n,i}$ of matrix K_i [21]. Apparently

$$D_i = \frac{1}{4}(\lambda_{\max\oplus 1,i} + \lambda_{\max\oplus 2,i}) \qquad (4)$$

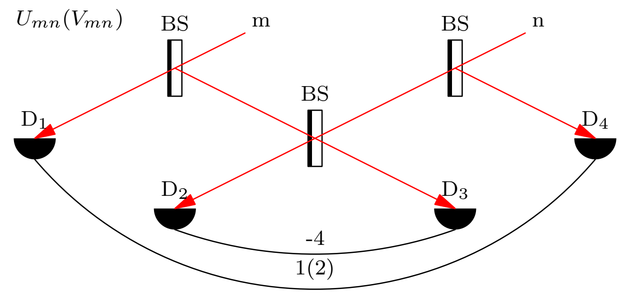

FIG. 1. (Color online) Setup for linear-optical measurement of U_{mn} (V_{mn}) observable. The setup consists of 50:50 beam splitters (BS) and conventional detectors (D). The possible measurement outcomes for U_{mn} (V_{mn}) are -4 if D_2 and D_3 click simultaneously as we witness singlet projection or 1 (2) if coincidence is registered D_1 and D_4 (this measures I). The device works with probability 0.5.

and $W_i = M_{1,i}^2 - M_{2,i} = 4\lambda_{\max,i}D_i - \lambda_{\max\oplus 1,i}\lambda_{\max\oplus 2,i}$, where $\oplus$ stands for sum modulo 3. It is apparent that for $\lambda_{m,i} \geq 0$ the witness W_i fulfills the necessary condition. This is because for positive λ_i the vanishing of GQD given in Eq. (4) implies $\lambda_{\max\oplus 1,i}\lambda_{\max\oplus 2,i} = 0$. Thus, the nullity of witness W_i is necessary for $D_i = 0$.

To demonstrate that W_i is a sufficient witness of classical correlations it we have to show that

$$\lambda_{\max\oplus 1,i}\lambda_{\max\oplus 2,i} = \lambda_{\max,i}(\lambda_{\max\oplus 1,i} + \lambda_{\max\oplus 2,i}) \qquad (5)$$

holds only if both sides are zero. Since $\lambda_{\max,i} \geq \lambda_{n,i}$ for $n = 0, 1, 2$ the equality is possible only if one of $\lambda_{n,i} = 0$, say $\lambda_{\max\oplus 2,i} = 0$, but this implies that $\lambda_{\max\oplus 1,i}\lambda_{\max\oplus 2,i} = 0$ and the only possibility for the equality to be satisfied is to have $\lambda_{\max\oplus 1,i} = 0$. Since $\lambda_{\max\oplus 1,i} = 0$ and $\lambda_{\max\oplus 2,i} = 0$ we must have $D_i = 0$, because D_i it is directly proportional to sum of those eigenvalues. Thus, our witness is a sufficient witness of quantum discord.

The witness W_A (W_B) is applicable as long as K_A (K_B) is positive semidefinite. Since, $\vec{x}\vec{x}^\dagger$ or $\vec{y}\vec{y}^\dagger$ is positive semidefinite, the requirement for K_i reduces to requiring $TT^\dagger$ to be positive semidefinite. This obviously happens, e.g., for symmetric states $\rho_{AB} = \rho_{BA}$ which include Bell-diagonal states and other states like symmetrically amplitude-damped or phase-damped Bell states [17, 22]. However, we also get $K_i \geq 0$ if T is diagonal which can always be assured by local unitary transformations on systems A and B [14]. Hence, our classicality witness can be considered universal up to local unitary operations. Moreover, since the symmetric geometric discord vanishes $D_s = 0$ only if $D_A = D_B = 0$ [20], for checking if the state exhibits classical-classical correlations $W_s = W_A + W_B$ should be used.

Linear-optical implementation. Up to this point we did not mention how to measure the CW. The proposed measurement procedure is similar to the proposal for direct measurement of quantum discord introduced by Jin

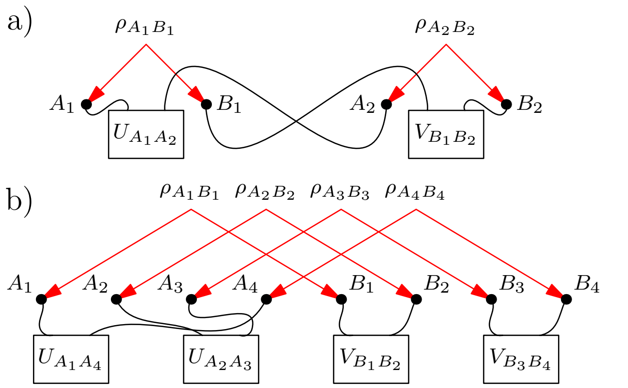

FIG. 2. (Color online) Setup for linear-optical measurement of moment (a) M_1 and (b) M_2. The setup works with probability 1/4 (1/16) for measuring M_1 (M_2). Note that the measurement of M_1 and M_2, and thus W, can be conducted in parallel if one has access to six copies of the input state ρ.

*et al.*in [21] and it resembles the so-called entanglement swapping [23].

In our case we need only first two moments of matrix K_i, and we need a fewer copies of ρ. The first moment is given as $M_{1,A} = \mathrm{Tr}(\vec{x}\vec{x}^\dagger + TT^\dagger)$, where we fixed without loss of generality $i = A$, and requires two copies of ρ to be measured. This is because $M_{1,A}$ can be also expressed [21] as

$$M_{1,A} = \mathrm{Tr}[(U_{A_1A_2} \otimes V_{B_1B_2})(\rho_{A_1B_1} \otimes \rho_{A_2B_2})], \quad (6)$$

where

$$U_{A_mA_n} = -4P^-_{A_mA_n} + I_{A_mA_n},\ V_{B_mB_n} = U_{B_mB_n} + I_{B_mB_n}. \quad (7)$$

Both U and V are given in terms of singlet projections $P^-_{i_mi_n} = |\Psi^-\rangle_{i_mi_n}\langle\Psi^-|$ and $|\Psi^-\rangle_{i_mi_n} = (|0\rangle_{i_m}|1\rangle_{i_n} - |1\rangle_{i_m}|0\rangle_{i_n})/\sqrt{2}$ with $i = A, B$ and $m, n = 1, 2$. However, for measuring $M_{2,A}$ we need four copies ρ, since

$$M_{2,A} = \mathrm{tr}[(U_{A_1A_4} \otimes U_{A_2A_3} \otimes V_{B_1B_2} \otimes V_{B_3B_4}) \times(\rho_{A_1B_1} \otimes \rho_{A_2B_2} \otimes \rho_{A_3B_3} \otimes \rho_{A_4B_4})]. \quad (8)$$

The projections P^- and I can be implemented using linear optics (see Fig. 1). The operators $U_{A_mA_n}$ and $V_{B_mB_n}$ are local two-qubit operators acting on the respective subsystems. Having four copies of ρ each moment can be estimated in a single coincidence measurement. However, if six copies are available, the CW from Eq. (3) could be evaluated in single measurement. Multiphoton experiments are demanding but feasible (for review see Ref. [24]). Nevertheless, with a few modifications the measurement can be implemented as presented below.

By choosing orthogonal photon polarization states as z-basis for subsystems constituting ρ we can implement the above-introduced witness from Eq. (3) in a linear-optical system. Our proposal for the experimental setup

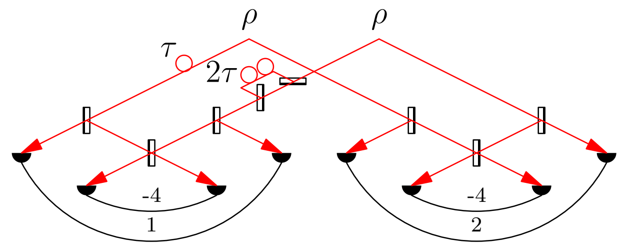

FIG. 3. (Color online) Optical setup for measuring M_2. When the delay lines are removed, the setup measures M_1 as in Fig. 2a. At time $t = 0$ and $t = \tau$ two photon pairs ρ enter the setup. When $t = 2\tau, 3\tau$ no new photons appear and only coincidences in the left part of the setup are witnessed. This part consists of U measurement, where one photon is always delayed by τ and the second one randomly by 2τ or 0. The second delay works with probability of 0.5. There is no delay in the right part of the setup measuring V. In the successful cases $\rho_{A_1B_1}$ and $\rho_{A_2B_2}$ enter the setup at $t = 0$ and $V_{B_1B_2}$ is measured. At the same time photons in the left part are delayed, the first from the left by τ and the second by 2τ (with probability 1/4), so at $t = 0$ there is no photon detected in U. Photon pairs $\rho_{A_3B_3}$ and $\rho_{A_4B_4}$ enter the setup at $t = \tau$ and $V_{B_3B_4}$ is measured without delay. In the left part, the first photon is delayed by τ, but the second one is not affected (with probability 1/4). At the same time $U_{A_1A_4}$ is measured. Since there are no photons added at $t = 2\tau$ only $U_{A_2A_3}$ is measured (for a photon provided at $t = 0$ and delayed by 2τ and a photon provided at $t = \tau$ delayed by τ). The success rate of the procedure is 1/256. The 1/16 drop in comparison to Fig. 2b is caused by applying probabilistic delay. The whole sequence is repeated until a good estimate of M_2 is obtained. No photons enter the setup at $t = 3\tau$ to not affect the next iteration.

consists of standard optical elements, i.e., 50:50 beam splitters (BS) and photon detectors. The implementation requires up to four elements from Fig. 1. This means using simultaneously up to four photon pairs in state ρ as shown in Fig. 2.

The U and V can be measured using the device shown in Fig. 1, where with probability 1/4 both photons pass unchanged through the pair of the 50:50 BS. Next, if both photons are detected by the D_1 and D_4, we measure the part of the operator U or V proportional to I. Alternatively, with probability 1/4, both photons reflect on the BSs and overlap on the central 50:50 BS. After overlapping on the BS the state ρ is projected onto a singlet state by witnessing coincidence in D_2 and D_3. A setup that can be used for measuring $M_{1,i}$ ($M_{2,i}$) consists of one (two) V_{mn} and one (two) U_{mn} element(s) arranged as shown in Fig. 2a(b). However, obtaining valid coincidences is probabilistic and thus the success rate of measuring M_1 is 1/4 and for M_2 it reaches 1/16. In spite of loosing many photons measuring W_i is expected to be much faster than full 2-qubit *quantum tomography* (QT) which requires rotating a number of polarization plates providing 16 measurement configurations, where each rotation takes a few seconds. During this time the

measurement setup from Fig. 2 would accumulate enough data to estimate W_i.

The main difficulty in implementing the setup outlined in Fig. 2 is working with large number of photons. The photon pairs should exhibit quantum correlations, thus should be produced, e.g., in spontaneous parametric down-conversion of light of a certain degree of depolarization. Since the pairs are produced by a random process, the coincidence rate in our experimental proposal can be very low. There are of course further challenges. Depending on the approach of producing the input states, one may face the problem of indistinguishably of the photon pairs, which causes the results to differ from the expected one, but methods of circumventing this effect can be found, e.g., in [24]. Moreover, the theoretical prediction is reached if all the pairs perfectly overlap on the corresponding BSs. This condition is easy to be satisfied for one pair of photons, however the amount of work increases with the number of photons. To achieve this all the optical elements would require active stabilization. Nevertheless, we can reduce complexity of the optical setup by using temporal separation between photon pairs ρ as shown in Fig. 3. The success rate of the measurement of M_2 from Fig. 3 is 1/256 instead of 1/16 as in the setup from Fig. 2b since it requires applying probabilistic delay for measuring U. When all the delay lines in Fig. 3 are removed, the setup corresponds to the one shown in Fig. 2a and can be used for direct measurement of M_1. By taking account for the pauses in providing two pairs of photons at $t = 2\tau, 3\tau$ (we only allow every second two pulses from the pulse train to enter the setup by using a pulse picker) the efficiency of the setup drops by a factor of 1/2. Thus, for perfect detectors, the useful coincidence count rate would be limited to 1/512 of the brightness of the source. This effectively changes the repetition rate of the source from 80 MHz to 160 kHz. Moreover, this rate is further reduced by a factor of 1/2 because of the 50 ns dead time of the detectors. Finally, we arrive at 80 kHz effective repetition rate of the source. In two-crystal type-I geometry (so-called Kwiat source [25]), the probability of generating an entangled photon pair from a single pump pulse is about 1/10. Thus, creating two pairs independently occurs with probability 1/100. Assuming perfect detectors, we can expect the photons generated with a pulsed laser at repetition rate of 80 kHz to cause the 8 photons (two pairs generated at $t = 0$ and other two at $t = \tau$) to arrive at the designated places 800 times per second. In reality this number would be much lower due to finite efficiency of the detectors. For realistic detector efficiency of 0.75 (Perkin-Elmer single photon counting modules operating at wavelength of 700 nm), the number of coincidences per second would reach about 80. This number would be further reduced by a few percent by unavoidable imperfections of the setup. Nevertheless, the final expected number of detection events for M_2 is of order of tens coincidences per second which would allow us to measure M_2 in less than a minute. Measuring M_1 would be much faster because of the simpler structure of coincidences, no pulse picking, and lack of probabilistic delay. The original setup for measuring M_2 outlined in Fig. 2b would also not suffer from such a low coincidence rate as the one shown in Fig. 3. However, the later alternative is more feasible and can be successfully implemented, e.g., in our laboratory.

Conclusions. We discovered a novel CW (witness of GQD) which is universal up to local choice of coordinate system. Our witness will work as long as the K_i matrix is positive semidefiite, which includes, e.g., symmetric states like Bell-diagonal or symmetrically-damped states [17, 22]. Therefore, our CW is more powerful than the ones reported recently by others [8–12]. In this respect can our CW compared with the one introduced by Zhang *et al.*in [13] which also requires four copies of the input state, but is not appropriate for optical implementation. Our approach is less complex than full QT, since it only requires 2 measurements, if 4 copies of ρ are available [either spatially (see Fig. 2) or time separated (see Fig. 3)], instead of 16 for the QT [26–28]. Furthermore, the proposed measurement setup is expected to be faster than the direct QT, since it would provide the outcome within a minute. Let us note that if the one had access to 6 copies of ρ, the witness W could be estimated in a single measurement in contrast to the simpler experiment described in [10], where the CW was not universal and its estimation involved measuring three quantities. Furthermore, our approach can be extended to directly measure GQD as discussed in Ref. [21]. We expect the simple mathematical expression for the CW to have a deeper physical meaning since it strongly resembles the expression for variance where role of the stochastic variable is played by eigenvalues of K_i and might be related to quantum phase transitions.

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[1] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).
[2] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Lett. A **23**, 880 (1969).
[3] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).
[4] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[5] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[6] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. **84**, 1655 (2012).
[7] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008).
[8] G. Passante, O. Moussa, D. A. Trottier, and R. Laflamme, Phys. Rev. A **84**, 044302 (2011).
[9] R. Auccaise, J. Maziero, L. C. Céleri, D. O. Soares-Pinto, E. R. deAzevedo, T. J. Bonagamba, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Phys. Rev. Lett. **107**, 070501 (2011).
[10] G. H. Aguilar, O. Jiménez Farías, J. Maziero, R. M. Serra, P. H. Souto Ribeiro, and S. P. Walborn, Phys. Rev. Lett. **108**, 063601 (2012).
[11] R. Rahimi, and A. SaiToh, Phys. Rev. A **82**, 022314 (2010).
[12] J. Maziero and R. M. Serra, Int. J. Quant. Inf. **10**, 1250028 (2012).
[13] Chengjie Zhang, Sixia Yu, Qing Chen, C. H. Oh, Phys. Rev. A **84**, 032122 (2011).
[14] S. Luo and Q. Zhang, J. Stat. Phys. **136**, 165 (2009).
[15] A. Peres, Phys. Rev. Lett.**77**, 1413 (1996).
[16] M. Horodecki, P. Horodecki, R. Horodecki, Phys. Lett. A **223**, 1 (1996).
[17] B. Horst, K. Bartkiewicz, A. Miranowicz, arXiv:1301.2969.
[18] B. Bylicka and D. Chruściński, Phys. Rev. A **81**, 062102 (2010).
[19] B. Dakić,V. Vedral, and Č. Brukner, Phys. Rev. Lett. **105**, 190502 (2010).
[20] A. Miranowicz, P. Horodecki, R. W. Chhajlany, J. Tuziemski, J. Sperling, Phys. Rev. A **86**, 042123 (2012).
[21] J. Jin, F. Zhang, C. Yu and H. Song, J. Phys. A: Math. Theor. **45**, 115308 (2012).
[22] Ş. Özdemir, K. Bartkiewicz, Y. Liu, and A. Miranowicz, Phys. Rev. A **76**, 042325 (2007).
[23] M. Żukowski, A. Zeilinger, M. A. Horne, A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[24] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Rev. Mod. Phys. **84**, 777 (2012).
[25] P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Phys. Rev. A **60**, R773 (1999).
[26] J.-S. Xu, C.-F. Li, C.-J. Zhang, X.-Y. Xu, Y.-S. Zhang, and G.-C. Guo, Phys. Rev. A **82**, 042328 (2010).
[27] J.-S. Xu, X.-Y. Xu, C.-F. Li, C.-J. Zhang, X. Zou, and G.-C. Guo, Nat. Commun. **1**, 7 (2010).
[28] D. O. Soares-Pinto, L. C. Céleri, R. Auccaise, F. F. Fanchini, E. R. deAzevedo, J. Maziero, T. J. Bonagamba, and R. M. Serra, Phys. Rev. A **81**, 062118 (2010).